# Dynamics of driven interfaces near isotropic percolation transition

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We consider the dynamics and kinetic roughening of interfaces embedded in uniformly random media near percolation treshold. In particular, we study simple discrete ''forest fire'' lattice models through Monte Carlo simulations in two and three spatial dimensions. An interface generated in the models is found to display complex behavior. Away from the percolation transition, the interface is self-affine with asymptotic dynamics consistent with the Kardar-Parisi-Zhang universality class. However, in the vicinity of the percolation transition, there is a different behavior at earlier times. By scaling arguments we show that the global scaling exponents associated with the kinetic roughening of the interface can be obtained from the properties of the underlying percolation cluster. Our numerical results are in good agreement with theory. However, we demonstrate that at the depinning transition, the interface as defined in the models is no longer self-affine. Finally, we compare these results with those obtained from a more realistic reaction-diffusion model of slow combustion. [S1063-651X(98)08708-X]

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## I. INTRODUCTION

Interfaces embedded in random media have received a considerable amount of interest recently. Such diverse phenomena as pinning of flux lines in superconductors, dynamics of flame fronts in paper, and imbibition all contain interfaces propagating in random media with quenched noise [1]. For many such cases, an equation of motion for the $d$-dimensional height variable $h(\vec{r},t)$ can be written in the form

$$\frac{\partial h(\vec{r},t)}{\partial t} = \nu \nabla^2 h(\vec{r},t) + \frac{1}{2}\lambda |\vec{\nabla} h(\vec{r},t)|^2 + F + \eta(\vec{r},h), \quad (1)$$

where $F$ is the driving force and the noise term $\eta$ represents quenched disorder and is sufficiently short ranged.

The behavior of driven interfaces near the depinning transition $F \rightarrow F_c$ at which the interface ceases to propagate and its average velocity $v$ approaches zero has turned out to be nontrivial. In particular, there are two important universality classes that many different models of interface dynamics fall into at the depinning transition, namely, the isotropic depinning (ID) or the directed percolation depinning (DPD) cases [1–3]. Roughly speaking, models whose microscopic dynamics is isotropic belong to the ID universality class and those with spatial anisotropy to the anisotropic universality classes, of which perhaps the most common one is the DPD case [3]. These universality classes can be distinguished by the values of the scaling exponents associated with the interface near the transition as well as the behavior of the nonlinear term $\lambda$ in the equation of motion for the interface. For the ID case, $\lambda$ is kinetically generated ($\lambda \sim v$) and vanishes at the transition, while for the DPD case this is no longer true.

In this work we report the results of extensive numerical simulations of some simple ''forest fire'' lattice models [4] where an interface propagates in a uniformly random background of reactants with an average concentration $0 < c < 1$. This is an interesting special case of a motion of an interface through a background medium of quenched noise, with the additional feature that there is an underlying *isotropic percolation transition* at some finite density $c^*$. Below $c^*$, the interface becomes pinned due to the percolation transition and one may expect different features to arise in this class of problems, which we call here *isotropic percolation depinning*. There is little work on the dynamics of interfaces in such isotropic lattice models, in particular near percolation [1,5]. These type of models are also interesting from the point of view of recent theoretical [6–8] and experimental [9,10] studies of dynamics of slow combustion in random media.

Our results indeed reveal interesting and complex behavior in the dynamics of the interface. Above the depinning transition for $c > c^*$, the kinetic roughening of the interface is found to be described asymptotically by the Kardar-Parisi-Zhang (KPZ) [11] universality class as described by Eq. (1) with annealed, Gaussian noise. Results consistent with the KPZ universality class were also found in the simulations of Refs. [6,7] of a more realistic continuum model of slow combustion. On approaching the percolation transition of the underlying lattice, $\lambda$ seems to decrease since the nonlinear term is kinetically generated in the present case. We find that in

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this regime, there is a different early-time behavior. We show that in this case the *global* scaling exponents characterizing the kinetic roughening of the interface can be obtained by utilizing results of percolation theory. In particular, this means that these exponents are completely determined by the properties of the percolation cluster and the continuum description of Eq. (1) must break down. Furthermore, we show that the interface at $c^*$ as defined in the models is no longer self-affine, but seems to show *multiscaling* since roughness exponents as measured numerically from different correlation functions differ [12].

The results from the discrete model are compared and contrasted with those obtained for a continuum phase-field model of slow combustion introduced and studied in Refs. [6–8]. We find that at high concentrations well above $c^*$, the two models display qualitatively similar behavior. However, as $c \to c^*$, the kinetic roughening of the interface is different in the two models, in that there is no evidence of crossover in the continuum model. We show through an analytic argument that this is essentially due to the divergence of the width of the front in the continuum model and can be understood in the framework of mean-field theory.

The remainder of this paper is organized as follows. In Sec. II the model is introduced and characterized in detail. In Sec. III the results of extensive Monte Carlo simulations in two and three spatial dimensions are presented. Also, a theory to explain the observed crossover in the dynamics is developed in this section. A comparison between the discrete and continuum models is carried out. Finally, in Sec. IV we conclude and discuss our results.

## II. ''FOREST FIRE'' MODELS

We consider the following simple forest fire (FF) cellular automaton models [4] on square and simple cubic lattices in two and three spatial dimensions, respectively. The status of each lattice site can be one of the following: (i) an empty site, (ii) a site occupied by an unburned tree, (iii) a site occupied by a burning tree, and (iv) a site occupied by a burned tree. Initially, a fraction $c(0<c<1)$ of the sites are occupied by a tree. The initial distribution of trees is uniformly random with no spatial correlations. In the two-dimensional (2D) case the lattice is of length $L$ in the $x$ direction with periodic boundary conditions and $L'$ in the $y$ direction with free boundary conditions. In the 3D case the lattice is of length $L$ also in the $z$ direction. Unless otherwise stated, $L' \gg L$ [13].

The front propagation is initiated at $t=0$ by igniting all the trees at the bottom of the lattice ($y=0$ in two dimensions and the $xz$ plane in three dimensions, respectively). The dynamics of the model is defined by the following set of rules: During one Monte Carlo time step, a burning tree ignites all the unburned trees in a fixed, finite region around it and becomes a burned tree. In this work, we consider the nearest-neighbor (NN) and next-nearest-neighbor (NNN) FF models in two dimensions, and the NN model in three dimensions. A burned tree will remain as such and new trees will not be generated during simulations, in contrast to several versions of this basic model that display self-organized criticality [14]. The position of the emerging interface $h(\vec{r},t)$ at column $\vec{r}$ is defined as the location of the highest burning tree *or* the highest burned tree, if there are no burning trees in that column [15]. We note that this definition is sufficient to make the interface single valued.

The continuum model for which we will also present some different results has been introduced and studied in Refs. [6–8]. Briefly, the model is based on a phase-field appraoch, utilizing a set of coupled partial differential equations (PDE's) describing the evolution of a thermal diffusion field $T(x,y)$ coupled to a random reactants concentration field $c(x,y)$. The interplay between thermal dissipation and reaction diffusion of heat generated by combustion determines the dynamics in the model. To study front propagation, the set of equations is discretized on a 2D lattice and solved numerically. In analogy with the FF model, the lattice sites are randomly filled with reactants (''trees'') that ''burn'' according to the kinetics defined by the PDE's. The main difference with respect to cellular automaton type of models is that not only is the dynamics more realistic, but that the effective range of interactions is in part determined by local combustion dynamics. Also, the interface in the continuum model is not sharp, but can be defined through the local maximum of the temperature field $T(x,y)$.

## III. RESULTS

In order to quantitatively characterize the kinetic roughening of the interface, we have considered the following quantities [1,16]. First, the *global* width $w(c,t,L)$ of the interface is defined by

$$w^2(c,t,L) \equiv \overline{\langle [h(\vec{r},t) - \overline{h(\vec{r},t)}]^2 \rangle}, \tag{2}$$

where the overbar denotes a spatial average over the system of size $L$ and angular brackets denote configuration averaging. Correspondingly, the *local* width of the interface $w_\ell(c,t)$ can be defined as

$$w_\ell^2(c,t) \equiv \langle \langle [h(\vec{r},t) - \langle h(\vec{r},t) \rangle_\ell]^2 \rangle_\ell \rangle, \tag{3}$$

where the notation $\langle\ \rangle_\ell$ now denotes spatial averaging over all subsystems of size $\ell$ of a system of total size $L$. For growing self-affine interfaces, both the global and local widths satisfy the Family-Viscek scaling relation [17] and have asymptotic behavior given by

$$w^2(t,L) \sim \begin{cases} t^{2\beta} & \text{for } t \ll L^z \\ L^{2\chi} & \text{for } t \gg L^z, \end{cases} \tag{4}$$

and correspondingly for $w_\ell^2(t)$. The quantities $\beta$ and $\chi$ define growth and roughness exponents, respectively, and $\chi = \beta z$ [18]. We note that in addition to using the width, scaling exponents can be obtained by using the height-height difference correlation function

$$C(r,t) = \overline{\langle \delta h(\vec{r}_0,t_0) - \delta h(\vec{r}_0+\vec{r},t_0+t)]^2 \rangle}, \tag{5}$$

with $\delta h \equiv h - \overline{h}$, in the appropriate regimes [1].

### A. Dynamics of one-dimensional interfaces

In the FF models, the emerging fire sweeps through all the sites that are connected by the nearest- or next-nearest-

neighbor rule throughout the system. The front motion can thus be sustained only for lattices whose average concentration $c$ is at or beyond the percolation treshold of a 2D square lattice, which is known to be $c^*\approx 0.592\,746$ and $c^*\approx 0.407\,254$ for the NN and NNN cases, respectively [19]. Thus the pinning of the interface below $c^*$ is a direct consequence of the static percolation transition and we call this phenomenon *isotropic percolation depinning* (IPD) here. There are two competing length scales in the problem, namely, the correlation length associated with the percolation transition $\xi(c)$ and the lateral correlation length of the moving interface $\xi_{||}(t)$ that grows like $t^{1/z}$ [20]. In the vicinity of $c^*$

$$\xi(c)\sim(c-c^*)^{-\nu}, \tag{6}$$

where $\nu$ defines the (static) correlation length exponent and $\nu=4/3$ in 2D percolation [19].

In the regime where pinning effects can be neglected, it has been demonstrated that the quenched noise in Eq. (1) crosses over to thermal noise [1]. In the FF models, this sitation is realized well above $c^*$, where $\xi(c)$ is essentially of the order of the lattice constant and $\xi(c)\ll\xi_{||}(t)$ readily holds. Indeed, in this regime we find that the interface moves with a constant velocity and its global width roughens asymptotically as given by Eq. (4), with $\beta\approx 1/3$ and $\chi\approx 1/2$, in accordance with the KPZ universality class. For example, for the NN model at $c=0.95$ we obtain $\beta=0.33(1)$ for $L=20\,000$ and $\chi=0.50(2)$ for a system size of $L=5000$. In Fig. 1 we plot $w(t)$ vs $t$ and the effective exponent $\beta_{\rm eff}(t)=\ln w(t)/\ln t$ that shows the asymptotic KPZ behavior. We would like to point out that there is an initial time regime where the width grows according to the uncorrelated random deposition model, with $w(t)\sim t^{1/2}$. For the NN model, this regime is long lived only for $c$ very close to unity [21].

The asymptotic KPZ behavior for $c>c^*$ is not unexpected since the velocity of the interface in the FF models is clearly tilt dependent, which generates the nonlinear term proportional to $\lambda$ in Eq. (1). In Fig. 2(a) we show the behavior of $\lambda$ as a function of $c$ above $c^*$ for the NN case. It has been calculated numerically by computing the average velocity $v$ of the interface as a function of the global tilt $m$ and fitting a parabola to it for $m\ll 1$ [1,5]. The interesting result is that $\lambda$ displays nonmonotonic behavior and seems to eventually *decrease* when approching the percolation transition. In fact, we expect that $\lambda(c)\to 0$ as $c\to c^*$ because the interface is eventually forced to propagate in the infinite percolation cluster, which is known to be self-similar and isotropic at the percolation threshold [22]. Since there is no preferred growth direction at $c^*$, the tilting of the interface should not affect the velocity of the interface any more.

This diminishing of $\lambda$ on approaching $c^*$ means that the nonlinear term in Eq. (1) becomes less and less important at early times where $\xi(c)\gg\xi_{||}$. For the continuum description to hold, however, the local slopes of the interface should also remain small. We have studied this numerically for various values of $c$ close to $c^*$, where two things can be observed for the behavior of the global width $w(t,L)$. First, the range of the late-time KPZ scaling regime becomes smaller in time as $c^*$ is approached from above. Second, another regime where well defined power-law scaling of $w(t)\sim t^{\beta^*}$ can be observed appears at earlier times. In Fig. 3(a) we show the behavior of the global width for a system of size $L=20\,000$ at $c=0.59\,275$ (NN model). We find that starting from early times, there is a scaling regime where the growth exponent $\beta^*=0.88(1)$. Simulations of the NNN model at $c=0.407$ give $\beta^*\approx 0.88$, correspondingly. In this regime parts of the interface become pinned by unburned regions on the lattice and the interface motion consists of large jumps, with large local slopes appearing. This behavior indicates that the interfaces may not be self-affine [12].

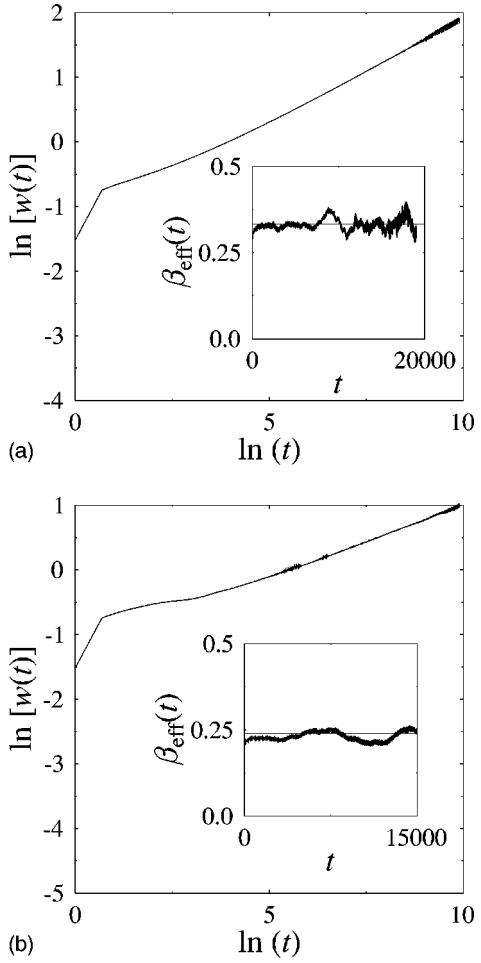


FIG. 1. (a) Global width $w(t)$ vs $t$ in two dimensions for the NN model with $c=0.95$ and $L=20000$. The inset shows the effective growth exponent $\beta_{\rm eff}(t)$ vs $t$, where $\beta_{\rm eff}(t)\equiv\ln w(t)/\ln t$. The exact KPZ value of $\beta=1/3$ is shown by the horizontal line. (b) Global width $w(t)$ vs $t$ for the 3D NN model, with $c=0.97$ and $L\times L=200\times 200$. In both cases averages were taken over 100 runs. The inset shows $\beta_{\rm eff}(t)$, with the horizontal value indicating the KPZ result $\beta\approx 0.24$.

The numerically observed crossover behavior can be formulated theoretically by assuming that it is induced by the underlying percolation transition. We write the following scaling form for the global width $w(c,t)$:

$$w(c,t)=\xi(c)f\left(\frac{t}{\tau_c}\right), \tag{7}$$

where $\tau_c$ denotes the crossover time to the KPZ regime and the scaling function $f(u)$ has the limits

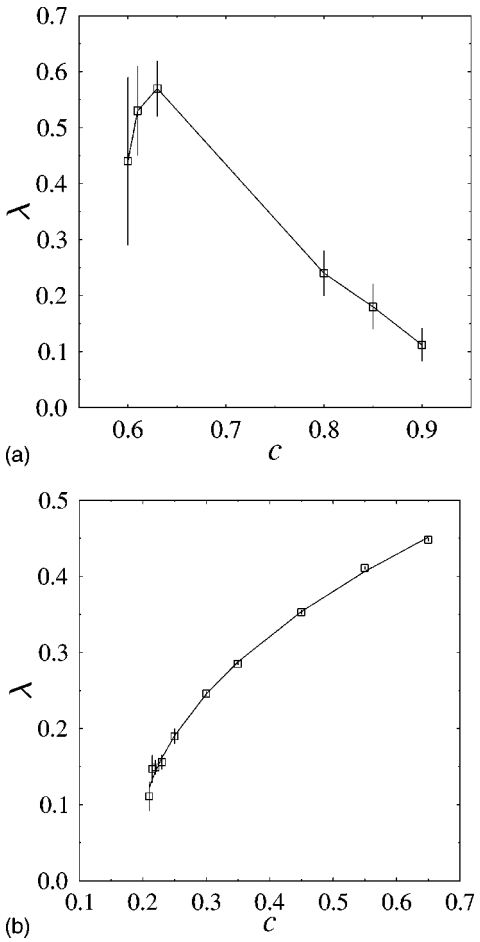


FIG. 2. (a) λ vs $c$ in two dimensions with $L=2000$ for the NN lattice model. The data were averaged over 1000 runs. (b) λ vs $c$ for the continuum model, with $L=200$.

$$f(u)\sim\begin{cases} u^{\beta^*} & \text{if } u\ll 1 \\ u^{\beta} & \text{if } u\gg 1. \end{cases} \tag{8}$$

Here $\beta^*\approx 0.88$ and $\beta=1/3$. Using Eq. (5) for $\xi(c)$ and assuming that $\tau_c(c)\sim(c-c^*)^{-\Delta}$ we find that best data collapse as shown in Fig. 4 is obtained for $\nu=1.3$ and $\Delta=1.65$. Taking $\nu=4/3$ and the dynamic exponent $z^*$ given by the exponent $d_{\min}\approx 1.13$ associated with the scaling of the minimum path distance [5,23], $\Delta=\nu z^*\approx 1.51$. Thus our numerical results are in good agreement with theory.

At $c^*$ where $\xi_{||}\ll\xi(c)$ for long times ($t\ll L^{z^*}$), the interface is pinned by clusters formed by the unoccupied sites and the quenched disorder dominates. The interface follows the ''edge'' of the infinite percolation cluster. The global roughness exponent $\chi^*$ can be then be directly deduced from the geometric properties of the percolation transition. In particular [5,1], $\chi^*=\nu_\perp/\nu_{||}$, where $\nu_\perp$ and $\nu_{||}$ are the perpendicular and parallel correlation length exponents of the critical percolation cluster, respectively. Since the percolation cluster in the FF model is isotropic and $\nu_\perp=\nu_{||}=\nu$, the roughness exponent $\chi^*=1$ *in all dimensions*. In this case, the exponent $z^*=d_{\min}\approx 1.13$, which leads to $\beta^*=\chi^*/z^*=1/d_{\min}\approx 0.88$, in excellent agreement with our simulations. These results indicate that the continuum description of Eq. (1) must break down at $c^*$ for the present IPD case [24].

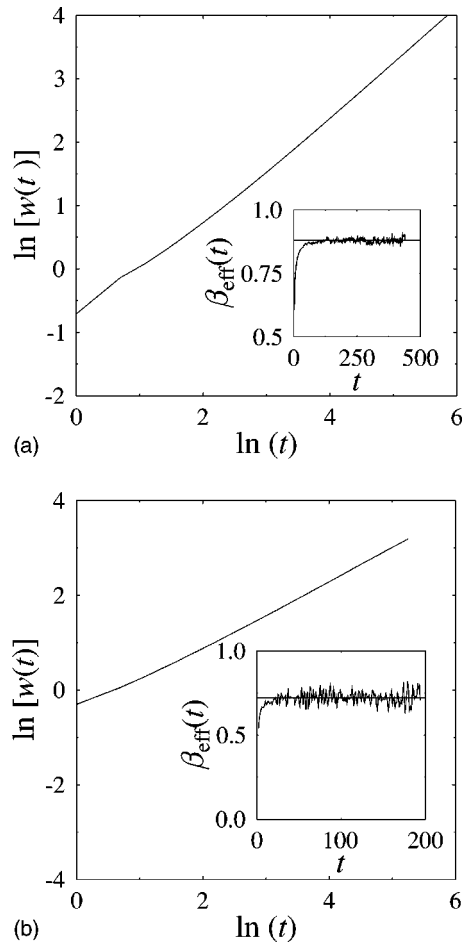


FIG. 3. (a) $w(t)$ vs $t$ in the 2D NN lattice model very close to the percolation transition ($c=0.592\,75$, $L=20\,000$). The inset shows the effective growth exponent $\beta_{\text{eff}}(t)$ and the horizontal line indicates the value 0.88. (b) $w(t)$ vs $t$ in the 3D NN lattice model very close to the percolation transition ($c=0.312$, $L\times L\times L=1100\times 1100$). The inset shows the effective growth exponent $\beta_{\text{eff}}(t)$ vs $t$ and the horizontal line indicates the value 0.72.

We have examined the interface roughness exponent $\chi$ numerically by studying the interface dynamics as close to $c^*$ as possible. We have computed the generalized $q$th-order height difference correlation functions

$$G^q(r,t)=\overline{\langle[h(r,t)-\overline{h}(t)]^q\rangle}\sim r^{q\chi_q} \quad \text{for } r\ll\xi_{||} \tag{9}$$

by running the simulation until the interface finally stops (for a finite system) and approximately traces out the edge of the percolation cluster. For a self-affine interface there is only one roughness exponent and thus $\chi=\chi_q$ for all $q=2,4,6,\ldots$. Our numerical results for a $L=2000$ system at $c=0.5928$ (NN model) give that $\chi_2=0.54(5)$, $\chi_4=0.29(3)$, and $\chi_6=0.21(2)$. This indicates that the interface associated with the percolation cluster *as defined in the model* is not self-affine at $c^*$. The reason is most likely that the overhangs in the front edge of the interface that follow the percolation cluster are removed. However, the scaling exponents for each higher-order correlation function that we have calculated seem to be very well defined, which is an indication of multiscaling similar to that seen in the longitudinal structure functions in the study of turbulence [25].

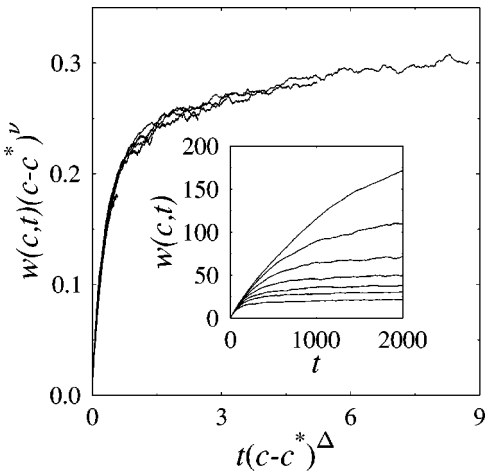


FIG. 4. Crossover scaling function $f(t/\tau_c)$ of the global width $w(c,t)$, as defined in Eq. (6). The unscaled data for different concentrations ($c=0.594$, 0.60, 0.605, 0.61, 0.615, 0.62, and 0.63, from top to bottom, and $L=1000$) are shown in the inset. The data collapse has been obtained using $\nu=1.3$ and $\Delta=1.65$. See the text for details.

We have also numerically verified the scaling of the average velocity $v(c)$ of the interface as a function of $c-c^*$ near the percolation threshold (see Fig. 5). It is expected to vanish as

$$v(c)=A(c-c^*)^{\theta}. \tag{10}$$

Our data for the NN model give $A\approx 1.14$ and our best estimate for the velocity exponent is $\theta=0.169(5)$ [the NNN model gives $\theta=0.17(5)$]. In order to check the consistency of this result, we note that there exists a well-known scaling relation between $\theta$, $z^*$, $\nu$, and $\chi^*$, namely [1],

$$\theta=(z^*-\chi^*)\nu. \tag{11}$$

By using the values $z^*=1.13$, $\chi^*=1$, and $\nu=4/3$ we obtain $\theta=0.173$. This is in very good agreement with our data.

### B. Dynamics of two-dimensional interfaces

The 3D lattice model that we have studied is a simple generalization of the 2D case to a simple cubic geometry. We only consider the NN case here. The behavior of the emerging surface near the percolation threshold is qualitatively similar to the 2D case. In particular, in the long time limit for $c>c^*$ the interface roughens in time with the growth exponent $\beta=0.24(2)$ as shown in Fig. 1(b), in excellent agreement with numerical solutions of the $d=2+1$ KPZ equation and various discrete models that belong to the KPZ universality class [1,26,27].

Closer to $c^*\approx 0.316$, we see the percolation-induced crossover. At $c=0.316$ we find numerically that the interface roughens with a growth exponent $\beta^*=0.72(5)$ [Fig. 3(b)]. Again, the exponents characterizing the interface can be obtained from the exponents of the critical percolation cluster. In particular, it is reasonable to assume that the global roughness exponent $\chi^*=1$ since the cluster is isotropic. Moreover, the minimum path exponent is known in $d=3$ to be $d_{\min}=1.38(2)$ and this determines the dynamic exponent

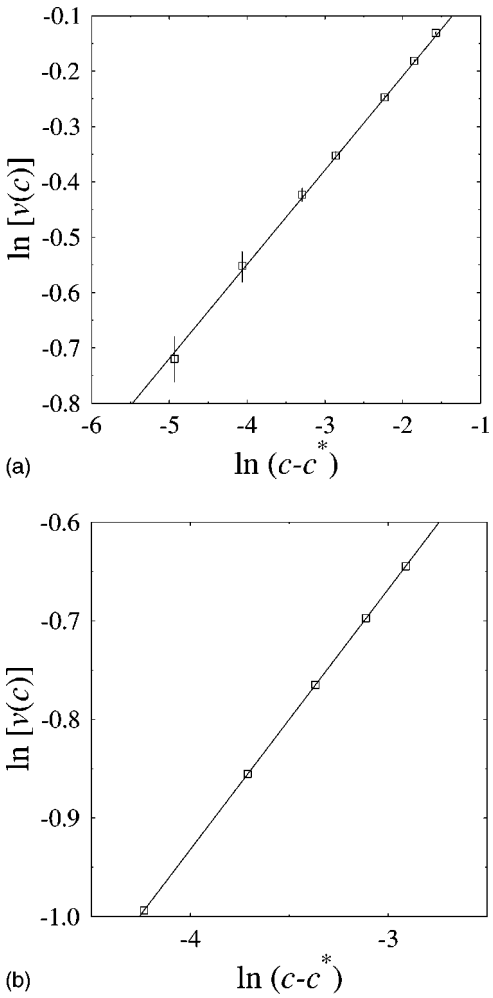


FIG. 5. (a) Scaling of the interface velocity $v$ vs $c-c^*$ for the 2D NN lattice model, with $L=2000$. The straight line shows the best fit to the data, with $\theta=0.169$ in Eq. (9). (b) Scaling of the interface velocity $v$ vs $c-c^*$ for the 3D NN lattice model, with $L\times L=100\times 100$. The straight line shows the best fit to the data, with $\theta=0.26$. The error bars here are smaller than the symbol sizes.

$z^*=d_{\min}$ [23]. As a consistency check, taken together with the result that $\chi^*=1$, this implies that, at depinning transition, $\beta^*=\chi^*/z^*\approx 0.724$. Our direct evaluation of $\beta$ agrees very well with this prediction. We expect again that the interface as defined in the 3D model is not self-affine at $c^*$; however, we have not computed $\chi^*$ numerically.

We have also calculated the velocity exponent and find $\theta=0.26(2)$. Using the scaling exponent relation $\theta=(z^*-\chi^*)\nu$ with $z^*=1.38$, $\chi^*=1$, and $\nu=0.88$ gives $\theta=0.33$, which is in reasonably good agreement with our data.

### C. Flame front propagation in the continuum model

We have compared the dynamics of interface of the lattice model with a more realistic continuum reaction-diffusion model of Refs. [6,7]. This model is a type of phase-field model that couples the evolution of a thermal diffusion field to a randomly distributed concentration field of reactants. The model couples the effects of thermal dissipation and diffusion to heat generated by combustion via an Arrhenius-activated reaction term. To study front propagation, the model is discretized on a 2D lattice and solved numerically.

In analogy with the FF model, the lattice sites are randomly filled with reactants (trees), with an average normalized concentration of $c \equiv \overline{c(x,y)}$. After ignition of the bottom row of reactants at $t=0$, the heat generated will ignite other occupied lattice sites around it and the local field $c(x,y)$ corresponding to the sites of the burning reactants will quickly approach zero as determined by the equations. A single-valued interface in the model is defined by the maximum of the temperature field $T(x,y)$ for each column $x$.

Previously, it was shown that that the kinetic roughening of the flame fronts generated by the continuum model belong to the thermal KPZ universality class [6,7]. In the limit of almost uniform background density, the KPZ description was also derived analytically from the set of equations for the model [7]. The main difference with respect to the FF lattice model was that even very close to the percolation threshold of the model $c^* \approx 0.20$, there was no evidence of percolation induced crossover. Also, the continuum model near $c^*$ gave results that were consistent with the mean-field theory of percolation, e.g., $\nu \approx 0.5$ and $\theta \approx 0.5$ [7].

In Fig. 2(b) we show the behavior of the nonlinear coefficient $\lambda$ for the continuum combustion model. Similarly to the FF lattice model, we find that $\lambda$ approaches zero for $c \rightarrow c^*$. However, unlike the the lattice model, no crossover behavior is observed as $c \rightarrow c^*$. This is explained as follows: From the mean-field analysis of Ref. [7], the leading front of the thermal field decays as

$$T_{\mathrm{MF}}(x) \sim e^{-x/l_D}, \tag{12}$$

where $l_D = D/v_m$ is the thermal diffusion length defining the range of effective interactions in the model and thus also the scale of the intrinsic thickness of the interface $w_{\mathrm{int}}$. The constants $v_m$ and $D$ are the mean interface velocity and thermal diffusion constant, respectively. Using the result that $v_m \sim (c-c^*)^{0.5}$, we conclude that $w_{\mathrm{int}} \sim (c-c^*)^{-0.5}$. On the other hand, in the mean-field percolation transition the correlation length scales as $\xi(c) \sim (c-c^*)^{-0.5}$. These results imply that the thickness of the interface has *the same divergence* as the correlation length, within which the crossover behavior should be observed. Thus everything that happens on length scales smaller than $w_{\mathrm{int}}$ will be smeared out. Therefore, due to the increasing thickness of the interface, the second regime at early times is never observed.

## IV. SUMMARY AND DISCUSSION

In this work we have studied the dynamics of interfaces in random media through Monte Carlo simulations of some discrete cellular automaton models of forest fires. We find that away from the depinning transition induced by the isotropic percolation transition of the underlying lattice, the kinetic roughening is asymptotically described by the Kardar-Parisi-Zhang [11] universality class. In the vicinity of the IPD transition, however, the behavior is found to be different. At the transition, the *global* roughness exponent $\chi^*$ and the growth exponent $\beta^*$ are completely determined by the geometric properties of the percolation transition, leading to the result that $\chi^* = 1$ and $\beta^* = 1/d_{\mathrm{min}}$ in all dimensions. We have verified this numerically for the exponent $\beta^*$ in the 2D and 3D cases. However, by computing the roughness exponent of the interface from different correlation functions, we find that the interface is no longer self-affine, but seems to indicate multiscaling. This is most likely due to the removal of overhangs in the way the interface is defined in the models.

A comparison between the lattice models and the more realistic model of Refs. [6–8] was made and qualitatively similar behavior was found at high concentrations. Interestingly, however, the two models displayed qualitatively different behavior for $c \rightarrow c^*$. In particular, the exponents compatible with the KPZ universality were shown to hold for all values of $c$ studied in Refs. [6,7]. We demonstrate that this can be understood on the basis of the mean-field nature of the percolation transition exhibited by the continuum model.

The models studied here are particularly interesting from the point of view of the recent experiments on slow combustion of paper [10,28]. In these experiments, asymptotic KPZ exponents were verified for driven interfaces. This is in complete agreement with all the models here well above percolation, as well as the DPD universality class. Near percolation, the assumption made on the basis of the earlier experiments by Zhang *et al.* [9] has been that DPD effects dominate [1]. However, the most recent experiments indicate [28] that the effective short-range exponents before KPZ asymptotics may not be well defined.

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[13] This is accomplished by employing a standard moving box routine, within which only a relatively narrow strip of the underlying reactant configuration around the interface needs to be stored in the computer memory. This allows us to simulate effectively infinite system sizes in the $y$ direction, i.e., $L'' \gg L$.

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[18] For some models it has been shown that the interfaces may not be truly self-affine and the saturated local width $w_{\ell} \sim \ell^{\chi_{\rm loc}}$, with $\chi \neq \chi_{\rm loc}$ [16]. Also, an anomalous temporal exponent can be defined through the difference $\beta - \chi_{\rm loc}/z$. However, we do not find evidence of this kind of anomalous scaling in the present models near $c^*$, since both the temporal and spatial scaling of the local and global widths (and the correlation functions for $q=2$) give the same exponents, i.e., $\beta \approx 0.88$ and $\chi \approx 0.54$.

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[20] Naturally the system size $L$ is the third length scale in the problem, but since we are interested in the crossover in time in $w(c,t,L)$, we assume here that $L$ is large enough.

[21] For the NNN model the asymptotic KPZ regime was found also, but in this case there is an additional transition to a smooth interface above $c_d \approx 0.54$. In the smooth regime, the interface advances with its maximum velocity $v=1$. This behavior is due to the directed percolation transition at $c_d$ associated with unoccupied sites on the lattice; see e.g., J. Kértesz and D. E. Wolf, Phys. Rev. Lett. **62**, 2571 (1989).

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[24] Incidentally, our estimates of $\beta^*$ and $\chi^*$ for the 2D case are very close to some of the numerical results obtained for the quenched Edwards-Wilkinson (QEW) equation that is given by Eq. (1) with $\lambda = 0$ [5,1,12]. However, in the QEW equation the interface is self-affine at and close to depinning [12] and its scaling exponents cannot be related to percolation theory.

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